ENERGY OF AN ASYMPTOTICALLY EQUIVALENT POINT DETONATION FOR THE DETONATION OF A CHARGE OF FINITE VOLUME IN AN IDEAL GAS

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The existence of a point detonation asymptotically equivalent to the detonation of a finite-volume charge (FVC) in an ideal gas follows from the laws of attenuation of blast waves at large distances from their place of origin [1, 2]; the parameters of the blast wave from the detonation of the FVC tend asymptotically to the parameters of the blast wave from the asymptotically equivalent point detonation (AEPD). This fact is consistent with the existing results of numerical calculations of various FVC detonation models [3-8].

The energy of the AEPD can be determined directly by numerical calculations of specific FVC detonation models [7, 8]. The expansion of the detonation products is rendered nonisentropic by secondary blast waves [3, 9, 10]; as a result, the exact analytical determination of the AEPD energy is an unsolvable problem at the present time. This predicament creates a natural requirement for the theoretical derivation of analytical estimates of the AEPD energy from just the initial parameters of the detonation products and the external gaseous medium. The work A_∞' done by the products of instantaneous detonation on air in the isentropic expansion of those products to the undisturbed air pressure has been adopted in [11] (p. 447) as the approximate value of the AEPD energy. However, this approach disregards all the principal factors governing the AEPD energy and thus leads to underestimated values of the latter both in relation to numerical calculations [7, 8] and in relation to the results obtained in [7] and the experimental data of [12].

In the present article we propose a procedure for determining the AEPD energy from the initial state of the detonation products and their coefficient of cubical expansion, and we obtain analytical upper and lower bounds for this energy. For elementary models of FVC detonation we derive simple analytical expressions for one of these bounds, which provide a good approximation for the AEPD energy. The accuracy of the approximations obtained under the assumption of isentropic expansion of the detonation products is limited by the influence of the secondary blast waves.

1. Let us suppose that a finite amount of energy E_0 is released instantaneously, i.e., a point detonation [13] occurs, at time $t = 0$ at a certain point O of an unbounded space occupied by a homogeneous gas at rest with pressure p_0 and adiabatic exponent γ. We choose an arbitrary closed control surface S bounding a finite volume V, which contains point O, and we determine the total energy flux across this surface.

At the initial time the total energy of the gas inside the volume V is equal to the sum of its internal energy $E = p_0V/(\gamma - 1)$ and the energy E_0 released in the thermal explosion. We conclude on the basis of the well-known results of numerical calculations of the point detonation problem [13] that in the limit $t \to \infty$ the gas in the volume V is at rest, its pressure is equal to the pressure p_0, and the total energy is equal to the energy E. Consequently, all the energy released in the point detonation flows across the control surface S in the time required for complete equalization of the pressure of the disturbed gas.

Making use of this characteristic property of a point detonation, we determine the AEPD energy for FVC detonation in a homogeneous gas at rest with pressure p_0, density ρ_0, and adiabatic exponent γ. Let the rest fuel mixture occupy a volume V_I bounded by a sphere of radius r_0, and let it be characterized by arbitrary distributions of the density ρ_C, specific heat of detonation Q, and adiabatic exponent γ_C with respect to the Lagrangian coordinate m. The Lagrangian coordinate m is interpreted as the mass of the gas contained in the volume of a sphere of radius r. Without committing to any specific model of detonation, we assume that the initial state of the detonation products is characterized by the velocity $u_1(m)$, pressure $p_1(m)$, density $\rho_1(m)$, and adiabatic exponent $\gamma_1(m)$.

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In solving the stated problem, we assume that the expansion of the detonation products is an adiabatic process. To simplify the subsequent calculations, we replace the initial fuel mixture by a fuel mixture with parameters ρ_C, Q^0, and γ_1 such that the initial state of the detonation products is identical with the state of the original mixture. By virtue of the law of energy conservation, the effective specific heat of detonation is $Q^0 = Q + (\gamma_1 - \gamma_C)p_0/(\gamma_C - 1)(\gamma_1 - 1)\rho_C$, and the corresponding effective detonation energy is

$$E^0 = \int_0^{m_0} Q^0 dm \tag{1.1}$$

(m_0 is the mass of the fuel mixture).

For the control surface we choose the surface S bounding the volume V_F of the detonation products after expansion to the pressure p_0. At the initial time the energy of the gas in V_F is equal to the sum of the initial energy of the detonation products

$$E_I = \int_0^{m_0} \left[\frac{u_1^2}{2} + \frac{p_1}{\rho_1(\gamma_1 - 1)} \right] dm \tag{1.2}$$

and the energy of the undisturbed gas contained in the volume V_F minus the volume V_I:

$$E_2 = \int_0^{m_0} \left(\frac{1}{\rho_F} - \frac{1}{\rho_1} \right) \frac{p_0}{\gamma - 1} dm \tag{1.3}$$

[$\rho_F(m)$ is the density of the expanded combustion products]. The energy of the expanded detonation products is

$$E_3 = \int_0^{m_0} \frac{p_0}{\gamma_1 - 1} \frac{dm}{\rho_F}. \tag{1.4}$$

On the basis of the energy conservation law we thus find that

$$E_\infty = E_I + E_2 - E_3 \tag{1.5}$$

expresses the energy transmitted across the selected control surface or across any other surface enclosing it. Consequently, E_∞ is the AEPD energy.

In the calculation of E_∞, because of the energy conservation law, the expression for E_I (1.2) can be replaced by the equivalent expression

$$E_I = E^0 + E_1, \tag{1.6}$$

where the effective internal energy of the fuel mixture is

$$E_1 = \int_0^{m_0} \frac{p_0}{\rho_c(\gamma_1 - 1)} dm. \tag{1.7}$$

By analogy with the energy E_∞, we determine the work A_∞ done on the external gaseous medium by the detonation products expanded to the pressure p_0:

$$A_\infty = E_I - E_3. \tag{1.8}$$

We say that A_∞ determines the work capacity of the detonation products. Comparing Eqs. (1.5) and (1.8), we find $E_\infty - A_\infty = E_2$. Consequently, E_∞ exceeds A_∞ by the energy of the undisturbed gas displaced by the expanded detonation products from the volume formed by the difference between V_F and V_I.

2. For a constant value of the adiabatic exponent γ_1 of the detonation products, Eqs. (1.5) and (1.8) are transformed as follows in accordance with (1.1)-(1.7):

$$E_\infty = E^0 - \frac{(\gamma-\gamma_1)(V_F - V_I)}{(\gamma-1)(\gamma_1-1)} p_0; \tag{2.1}$$

$$A_\infty = E^0 - \frac{V_F - V_I}{\gamma_1 - 1} p_0. \tag{2.2}$$

It follows from Eq. (2.1) that the inequality $E_\infty \gtrless E^0$ ($\gamma_1 \gtrless \gamma$) is associated with the difference between the properties of the detonation products and the external gaseous medium as expressed by different values of their adiabatic exponents.

We use the following equation, which is a direct consequence of Eqs. (2.1) and (2.2), to estimate a priori bounds of the energy E_∞:

$$E_\infty = \frac{\gamma_1-1}{\gamma-1} E^0 + \frac{\gamma-\gamma_1}{\gamma-1} A_\infty. \tag{2.3}$$

Invoking the inequality $A_\infty > 0$, which holds for $E^0 > 0$, we find the simple analytical bounds

$$E_\infty \gtrless \frac{\gamma_1-1}{\gamma-1} E^0 \quad (\gamma \gtrless \gamma_1), \tag{2.4}$$

which take into account the influence of secondary blast waves generated in FVC detonation on the energy E_∞ under the above-stated assumptions. The coefficient of E^0 in (2.4) has a simple physical significance: It is equal to the ratio of the effective energies of the products of instantaneous detonation with the adiabatic exponents γ_1 and γ and equal initial excess (gauge) pressures.

3. If entropy is assumed to be conserved in each individual particle of the expanding detonation products, the relation $\rho_1(m)/\rho_F(m) = [p_1(m)/p_0]^{1/\gamma_1(m)}$ holds, which enables us to determine E_∞' and A_∞' in the form

$$E_\infty' = A_\infty' + [p_0/(\gamma-1)] \int_0^{m_0} \{[(p_1/p_0)^{1/\gamma_1} - 1]/\rho_1\}\, dm; \tag{3.1}$$

$$A_\infty' = \int_0^{m_0} \left\{ \frac{u_1^2}{2} + \frac{p_1 - p_0}{\rho_1(\gamma_1-1)} - \frac{p_0}{\rho_1(\gamma_1-1)} \left[(p_1/p_0)^{1/\gamma_1} - 1\right] \right\} dm, \tag{3.2}$$

i.e., exclusively from the initial state of the detonation products and the external gaseous medium.

In particular, for the expanding products of an instantaneous detonation with constant initial values of the pressure p_1 and adiabatic exponent γ_1 we have

$$E_\infty' = \eta E^0, \quad A_\infty' = \alpha E^0. \tag{3.3}$$

The dimensionless characteristics of the FVC detonation (the energy coefficient η [7] and the coefficient α [11]) are functions of the dimensionless initial excess pressure $\Delta p_1 = (p_1 - p_0)/p_0$:

$$\eta = 1 - \frac{\gamma-\gamma_1}{\gamma-1} \varphi(\Delta p_1); \tag{3.4}$$

$$\alpha = 1 - \varphi(\Delta p_1) \quad (\varphi(\Delta p_1) = [(1+\Delta p_1)^{1/\gamma_1} - 1]/\Delta p_1). \tag{3.5}$$

Asymptotic representations for the coefficients η and α will be to our advantage in the ensuing investigation, viz.: for $\Delta p_1 \ll 1$

$$\eta(\Delta p_1) = \frac{(\gamma_1-1)\gamma}{(\gamma-1)\gamma_1} - \frac{\gamma_1-\gamma}{2\gamma_1(\gamma_1-1)} \Delta p_1, \quad \alpha(\Delta p_1) = \frac{\gamma_1-1}{\gamma_1} + \frac{\gamma_1-1}{2\gamma_1^2} \Delta p_1; \tag{3.6}$$

for $\Delta p_1 \gg 1$

$$\eta(\Delta p_1) = 1 - \frac{\gamma-\gamma_1}{\gamma-1} \Delta p_1^{-(\gamma_1-1)/\gamma_1}, \quad \alpha(\Delta p_1) = 1 - \Delta p_1^{-(\gamma_1-1)/\gamma_1}. \tag{3.7}$$

It follows from a quantitative analysis of relations (3.4) and (3.5), which were obtained on the assumption of isentropic expansion of the products of an instantaneous detonation, that the calculation of the energy E_∞' according to [11] undervalues this energy by ~10%, even for condensed high explosives. The amount of undervaluation attains 40-50% for fuel—oxygen and fuel—air mixtures. The increase in the entropy of the detonation products as a result of secondary blast waves causes the volume V_F of the expanded detonation products to increase. Since

$$A_\infty = A_\infty' - p_0 \Delta V_F/(\gamma_1 - 1), \quad E_\infty = E_\infty' - (\gamma - \gamma_1) p_0 \Delta V_F/(\gamma_1 - 1)(\gamma - 1) \tag{3.8}$$

in this case (ΔV_F is the increment in the volume of the expanded detonation products), we infer from the inequality $A_\infty/E_\infty < A_\infty'/E_\infty'$, which follows from (3.8), that the relative undervaluation of E_∞ when it is replaced by A_∞ is even more appreciable.

The density of the adiabatically expanded detonation products obeys the inequality $\rho_1/\rho_F > (p_1/p_0)^{1/\gamma_1}$, which enables us to obtain additional bounds for E_∞ on the basis of (1.2)-(1.5) and (3.1): $E_\infty \gtrless E_\infty'$ ($\gamma_1 \gtrless \gamma$). These bounds together with (2.4) yield important upper and lower bounds for E_∞:

$$\frac{\gamma_1 - 1}{\gamma - 1} E^0 \gtrless E_\infty \gtrless E_\infty' \quad (\gamma_1 \gtrless \gamma). \tag{3.9}$$

We now determine the AEPD energy for isentropically expanding products of a Chapman—Jouguet detonation. Since the propagation of the wavefront of such a detonation wave is a steady-state process, the entropy is identical for all particles of the detonation products. Consequently, the pressure $p_1(m)$ and the density $\rho_1(m)$ are related by the equation

$$p_1 = k\rho_1^{\gamma_1}, \tag{3.10}$$

where the constant k is determined from the conditions at the detonation wavefront as a compression shock with heat input [13]; it has the form

$$k = \frac{p_0}{\rho_0^{\gamma_1}(\gamma_1 + 1)} \left(\frac{\gamma_1}{\gamma_1 + 1}\right)^{\gamma_1} \frac{\left(1 + B + \sqrt{B^2 - \gamma_1^2}\right)^{\gamma_1 + 1}}{\left(B + \sqrt{B^2 - \gamma_1^2}\right)^{\gamma_1}} \quad \left(B = (\gamma_1^2 - 1) Q^0 \rho_0 p_0^{-1} + \gamma_1\right).$$

For the expanded detonation products we obtain the expression $\rho_F = (p_0/k)^{1/\gamma_1}$ from relation (3.10); substituting this expression in Eq. (3.1), we obtain

$$E_\infty' = \eta E^0. \tag{3.11}$$

Here

$$\eta = 1 - \frac{\gamma - \gamma_1}{(\gamma - 1) q} \left\{ \frac{\gamma_1}{B_1} \left(\frac{1 + B_1}{\gamma_1 + 1}\right)^{(\gamma_1 + 1)/\gamma_1} - 1 \right\}; \tag{3.12}$$

$$q = \rho_1 Q^0 (\gamma_1 - 1)/p_0, \quad B_1 = \gamma_1 + (\gamma_1 + 1) q + \sqrt{[\gamma_1 + (\gamma_1 + 1) q]^2 - \gamma_1^2}. \tag{3.13}$$

It follows from relations (3.10) that η has the following asymptotic representations in the limiting cases $q \ll 1$ and $q \gg 1$:

$$\eta(q) = \frac{(\gamma_1 - 1)\gamma}{(\gamma - 1)\gamma_1} + \frac{(\gamma - \gamma_1)(\gamma_1 - 1)}{3\gamma_1^2(\gamma - 1)} \sqrt{\frac{2\gamma_1}{\gamma_1 + 1} q} \quad (q \ll 1); \tag{3.14}$$

$$\eta(q) = 1 - \frac{(\gamma - \gamma_1)\gamma_1 2^{1/\gamma_1}}{(\gamma - 1)(\gamma_1 + 1)} q^{-(\gamma_1 - 1)/\gamma_1} \quad (q \gg 1). \tag{3.15}$$

On the basis of the energy conservation law, the initial dimensionless excess pressure of an instantaneous detonation is expressed as $\Delta p_1 = \rho_1 Q^0(\gamma_1 - 1)/p_0$ and, by virtue of (3.12), coincides with the effective dimensionless specific heat of detonation q. Comparing (3.7) and (3.15), we arrive at the conclusion that for $q \gg 1$, because of the inequality $0.942 < 2^{(1-\ln 2)/\ln 2} \ln 2 < \gamma_1 2^{1/\gamma_1}/(\gamma_1 + 1) < 1$, which holds for $\gamma_1 \geq 1$; the value of the energy coefficient for the expanding products of a Chapman—Jouguet detonation is greater (smaller) than the value of the same coefficient for the expanding products of an instantaneous detonation for $\gamma_1 < \gamma$ ($\gamma_1 > \gamma$). An additional numerical analysis of Eqs. (3.4) and (3.11) and of relations (3.6) and (3.14) shows that the foregoing conclusion as to the values of the energy coefficients for the two investigated detonation models is true for any values of $q > 0$. Moreover, the relative difference between the values of these energy coefficients for $\gamma_1 > 1.1$ does not exceed 3%.

The specific values obtained for the AEPD energies in the case of expanding products of a Chapman–Jouguet detonation exhibit good agreement (the relative discrepancy does not exceed 1%) with the values obtained in [8] for five compositions of a detonating gas mixture.

4. Let the state of the gas in the volume V bounded by the spherical surface of the blast wave at an arbitrary time after the start of expansion of the detonation products be characterized by the functions $p(m, t)$, $\rho(m, t)$, $u(m, t)$, and $\gamma(m)$ [$\gamma(m)$ is a piecewise-constant function, which is equal to γ_1 for the detonation products and is equal to γ for the gas disturbed by them]. Assuming that the expansion of the detonation products and the disturbed gas is an isentropic process, we find their total work capacity. We use the integral equation (3.2) for this purpose. Replacing the upper limit of integration m_0 in this equation by the Lagrangian coordinate m_2 of the blast wave and replacing the functions u_1, p_1, ρ_1, and γ_1 in the integrand by the respective functions u, p, ρ, and γ, we obtain the unknown work:

$$A = I_1 - I_2 - I_3, \quad I_1 = \int_0^{m_2} \left[\frac{u^2}{2} + \frac{p}{\rho(\gamma - 1)}\right] dm,$$

$$I_2 = \int_0^{m_0} \frac{p_0}{(\gamma_1 - 1)\rho_F(m)}\, dm, \quad I_3 = \int_{m_0}^{m_2} \frac{p_0}{(\gamma - 1)\rho_F(m)}\, dm. \tag{4.1}$$

The integral I_1 determines the total energy of the gas behind the blast wave. According to the energy conservation law, it is equal to the sum of the effective detonation energy E^0, the internal energy of the fuel mixture occupying the volume V_I, and the initial energy of the external gaseous medium in the volume V minus the volume V_I:

$$I_1 = E^0 + \frac{p_0 V_I}{\gamma_1 - 1} + \frac{p_0 (V - V_I)}{\gamma - 1}. \tag{4.2}$$

The integrals I_2 and I_3 express the residual energy of the detonation products expanded to the pressure p_0 and the gas disturbed by them at the given time. The condition of entropy conservation in each individual particle of the gas behind the blast wavefront permits I_2 to be written in the form

$$I_2 = \frac{p_0}{\gamma_1 - 1}\int_0^{m_0} \left(\frac{p_1}{p_0}\right)^{1/\gamma_1} \frac{dm}{\rho_1}. \tag{4.3}$$

The same condition can be used to replace the integration in space at the given time by integration along the path of the blast wavefront in the computation of I_3, so that

$$I_3 = \frac{p_0}{\gamma - 1}\int_{m_0}^{m_2} \left[\frac{p(m_2)}{p_0}\right]^{1/\gamma} \frac{dm_2}{\rho(m_2)}. \tag{4.4}$$

Taking Eqs. (4.1) and (4.2) into account, we then have

$$A(m_2) = A(m_0) - \frac{p_0}{\gamma - 1}\int_{m_0}^{m_2} \Phi\rho_0^{-1} dm_2. \tag{4.5}$$

Here $A(m_0) = A_\infty$, and the function Φ can be expressed as follows on the basis of the Hugoniot adiabat:

$$\Phi(\Delta p) = (1 + \Delta p)^{1/\gamma} \frac{(\gamma - 1)\Delta p + 2\gamma}{(\gamma + 1)\Delta p + 2\gamma} - 1 \tag{4.6}$$

[$\Delta p = (p_2 - p_0)/p_0$ is the relative excess pressure in the blast wave]. The radius r_2 of the blast wavefront is related to the Lagrangian coordinate m_2 by the equation $m_2 = m_0 + \delta(r_2^\nu - r_0^\nu)\rho_0/\nu$, where ν is a symmetry index (ν = 1, 2, 3 corresponds to planar, axial, and point symmetry), and $\delta = 2\pi(\nu - 1) + (\nu - 2)(\nu - 3)/2$. We introduce the dimensionless radius of the blast wavefront $R = r_2(p_0/\eta E^0)^{1/\nu}$. We denote its initial value by R_0 and transform (4.5) as follows:

$$A(R) = A(R_0) - \frac{\delta\eta E^0}{\gamma - 1}\int_{R_0}^{R} \Phi(\Delta p) R^{\nu-1} dR. \tag{4.7}$$

For simplicity we restrict the ensuing discussion to the case of expanding products of an instantaneous detonation with a constant initial distribution of the parameters. Then

$$A(R_0) = E^0[1 - \varphi(\Delta p_1)] \quad (\Delta p_1 = (\gamma_1 - 1)\nu/\delta R_0^\nu \eta(R_0)). \tag{4.8}$$

Passing to the limit $R_0 \to 0$ in (4.7) and allowing for the fact that, according to (4.8), $\lim\limits_{R_0 \to 0} A(R_0) = E^0$, we obtain the expression

$$A_{\mathrm{PD}} = E^0\left(1 - \frac{\delta}{\gamma - 1}\int_0^R \Phi(\Delta p) R^{\nu-1} dR\right), \tag{4.9}$$

which is valid for a point detonation. At large distances from the center of the FVC detonation, the work that can ultimately be done by the detonation products and the disturbed medium on the undisturbed medium must tend to its value for the AEPD. This condition implies that

$$\lim_{R\to\infty} [A(R)/\eta(R_0)] = \lim_{R\to\infty} A_{\mathrm{PD}}(R).$$

Since the right-hand side is a constant, which is uniquely determined by the parameters of the point detonation, the value of the left limit for FVC detonation does not depend on R_0; we denote this constant by A_*, whereupon we obtain

$$A_* = A(R_0)/\eta(R_0) - \frac{\delta E^0}{\gamma - 1}\int_{R_0}^{\infty} \Phi(\Delta p) R^{\nu-1} dR. \tag{4.10}$$

To evaluate it, we let $R_0 \to \infty$ in Eq. (4.10). We show that

$$\lim_{R_0\to\infty} \int_{R_0}^{\infty} \Phi(\Delta p) R^{\nu-1} dR = 0. \tag{4.11}$$

In fact, since the excess pressure of the products of instantaneous detonation obey

$$\Delta p_1 \ll 1 \quad (R_0 \gg 1), \tag{4.12}$$

and the excess pressure in the blast wave obeys $\Delta p < \Delta p_1$, the inequality $\Delta p \ll 1$ ($R_0 \gg 1$) holds. The latter enables us to use the relation $\Phi(\Delta p) = \beta\Delta p^3$ for the integrand $\Phi(\Delta p)$ and to represent the integral (4.10) in the form

$$\int_{R_0}^{\infty} \Phi(\Delta p) R^{\nu-1} dR = \beta(I_4 + I_5), \quad I_4 = \int_{R_0}^{R_1} \Phi(\Delta p) R^{\nu-1} dR,$$
$$I_5 = \int_{R_1}^{\infty} \Phi(\Delta p) R^{\nu-1} dR. \tag{4.13}$$

We interpret R_1 as the blast wavefront radius at which the well-known asymptotic expressions for $\Delta p(R)$ (see, e.g., [13]) begin to be valid:

$$\begin{aligned} &\Delta p(R) = a_1 R^{-0.5} && (\nu = 1), \\ &\Delta p(R) = a_2 R^{-0.75} && (\nu = 2), \\ &\Delta p(R) = a_3/R\sqrt{\ln R + a_4} && (\nu = 3) \end{aligned} \tag{4.14}$$

(a_i are constants).

For $R_0 \gg 1$, R_1 corresponds to the radius of the blast wavefront when it overtakes the rarefaction wave generated in the decay of an arbitrary discontinuity and reflected from the center of symmetry, so that

$$R_1 = cR_0^2 \tag{4.15}$$

(c is a constant).

Taking (4.12), (4.15), and the inequality $\Delta p < \Delta p_1$ into account, we obtain the upper bound for the integral $I_4 < [(\gamma_1 - 1)\nu/\delta]^3 c^\nu R_0{}^\nu$, so that $\lim\limits_{R_0\to\infty} I_4 = 0$.

In turn, the substitution of (4.14) in the integral I_5 yields the asymptotic expressions

$$\begin{aligned} &I_5 = 2a_1^3 R_1^{-0.5} && (\nu = 1), \\ &I_5 = 4a_2^3 R_1^{-0.25} && (\nu = 2), \\ &I_5 = 2a_3^3 (\ln R_1 + a_4)^{-0.5} && (\nu = 3). \end{aligned} \tag{4.16}$$

We infer from (4.16) that $\lim\limits_{R_0\to\infty} I_5 = 0$. We have thus proved the validity of Eq. (4.11). Consequently, $A_* = \lim\limits_{R_0\to\infty} [A(R_0)/\eta(R_0)]$. We pass to the limit by means of (4.8). As a result, $A_* = (\gamma - 1)E^0/\gamma$.

Finally, relation (4.10) yields the following expression for the integral of the entropy losses in the blast wave for FVC detonation:

$$\frac{\delta\eta(R_0)}{\gamma - 1}\int_{R_0}^{\infty} \Phi(\Delta p) R^{\nu-1}\, dR = \alpha(R_0) - \frac{\gamma - 1}{\gamma}\eta(R_0).$$

Letting the initial radius R_0 of the volume of combustion products tend to zero, we find the integral of entropy losses for a point detonation

$$\frac{\delta}{\gamma - 1}\int_0^{\infty} \Phi(\Delta p) R^{\nu-1}\, dR = \frac{1}{\gamma},$$

the form of which coincides with the integral obtained in [9] by an alternative procedure.

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GASDYNAMIC CHARACTERISTICS OF FLOWS IN PROBLEMS OF THE LAUNCHING OF INCOMPRESSIBLE PLATES BY DETONATION PRODUCTS

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There has been growing interest lately in analytical methods for the solution of one-dimensional gasdynamic problems involving the launching of incompressible plates [1-7]. This preoccupation stems from the relative simplicity of theoretical investigations and the feasibility of obtaining analytical solutions, identifying the principal gasdynamic characteristics of the generated flows, and both predicting and optimizing the gasdynamic possibilities of the analyzed launching configurations when the flow of detonation products is assumed to be isentropic and the launched plate is assumed to be incompressible.

The majority of the flow regions studied in [1-6] represent centered rarefaction waves, except that the centers of the waves can either be a part of or lie outside the analyzed region of flow of the detonation products, depending on the initial and boundary conditions of the problem. In this case the solutions can have regions where the families of rectilinear $(u \pm c)$-characteristics do not have a unique point of intersection (wave center), but form an envelope, which lies outside the investigated wave region. A similar situation arises, e.g., in the convergence of the characteristics in a compression wave. The difference is that the envelope in the latter case is situated in the wave region. An equation has been derived [7] for the envelope of the family of characteristics of a simple compression wave. In the present article we investigate a procedure for determining the envelope of the family of characteristics of a rarefaction wave. We analyze a method based on the equation for the envelope of a rarefaction wave for solving the planar one-dimensional isentropic gasdynamic equations.

One of the possible situations, which is associated with the occurrence of an envelope in the problem of the launching of a plate by detonation products is depicted in Fig. 1, which shows the trajectory of the plate 1, the envelope of the rarefaction wave 2, and the analyzed region I of flow of the detonation products. In this case the envelope is formed

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